Mathematical Documentation for the Balkh River Decision Support Tool

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1 Scope and Use of the Tool

To provide a framework for addressing the major challenges seen in Afghanistan, an Integrated Water Resources Management (IWRM) tool was developed. The tool is used to understand and improve the economic and food security performance of irrigation water use patterns in Afghanistan's Balkh River Basin. The IWRM seeks to enhance the capacity of Afghan stakeholders to assess the impacts of water availability and alternative water administration measures on the level and distribution of farm incomes. The IWRM also provides a means for Afghan policy makers to study policies that have an effect on crop prices, yields and costs and their implications in income at the farm in basin level. The IWRM tool brings together the best available crop and farm data in order to improve farm incomes and basin-wide food security.

The IWRM is an optimization model of the agricultural sector of the Balkh Basin. It ties together the hydrologic, agronomic, economic, policy and institutional dimensions of the Basin. Results associated with policies that would affect water allocation among 14 canals are presented for five important water shortage sharing arrangements that would affect farm income and wheat food security, and it has the flexibility to examine any alternative arrangement for sharing water shortages.

1.1 Description of the Canal System

Based on a canal schematic supplied by Mohammad Fahim Zaheer and MEW staff in July 2009, there are 14 separate canals in the Balkh Basin, and each canal area has a historical number of paikals[1] that are allocated to it during periods of full supply. The following table displays the name of each canal, its historical allocation under full supply (in paikals) and the area of land that can be irrigated by its historical allocation. A full water supply in the model is 5,253 paikals.

Table 1: Canal Characteristics, Balkh Basin, Afghanistan

Canal Name	**Historical Canal Allocation with Full Supply (Paikal)**	**Land Capacity (Ha)**
Aman Sahib	200	16,000
Nahr Shahi	560	44,800
Siagard	150	12,000
Balkh	70	5,600
Chemtal	164	13,120
Mushtaq	209	16,720
Abdulah	700	56,000
Dawlatabad	750	60,000
Charbulak	750	60,000
Faizabad	600	48,000
Murdian	332	26,560
Khanaqah	328	26,240
Aqcha	201	16,080
Mingajik	239	19,120
Basin Wide	**5,253**	**420,240**

2 Mathematical Documentation

The support tool makes use of a linear programming model in its optimization. The first step for each of the water allocation arrangements is that the total basin inflows are shared between the canals through the rules of the five different water sharing arrangements.

2.1 Water Shortage Allocation Arrangements

In this framework, when a full supply is available, each of the shortage sharing arrangments produce the same results, as each canal receives its historical allocation, which is the maximum amount of water that can be allocated to that canal. When less than a full supply is available, each of the arrangments shares water in the manner described in the following paragraphs. Each of the sets of formulas used to calculate the water allocations can be seen in row nine on the results page for each water sharing arrangement (labeled with the arrangement's name), with the exception of the User Inserted allocation. Let i be the set containing the 14 canals.

2.1.1 Upstream Priority

Under upstream priority, the most upstream canal takes a full allocation while the next lower canal takes its full allocation for any remaining water, and so on until the water runs out. This is performed by the minimum function:

$$\min(TotalWaterSupply, HistoricalWaterAllocation_{i=1}) \tag{1}$$

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[1] a paikal is an Afghan land-water unit, that equates to 80 hectares of irrigated land

For the most upstream canal. The subsequent canals are calculated as:

$$\min(TotalWaterSupply - \sum_{n=1}^{i-1} WaterAlreadyAllocated_i, HistoricalWaterAllocation_i) \tag{2}$$

These functions find the smallest value between the maximum water allocation for that canal and the amount of water that has not yet been allocated. As an example, to find the allocation for the fourth canal, we set $i = 4$. Therefore, equation 2 takes the form:

$$\min(TotalWaterSupply - \sum_{n=1}^{3} WaterAlreadyAllocated_i, HistoricalWaterAllocation_4)$$

Thus, the first argument of the minimum function becomes the total water supply minus the sum of the water that was allocated to the first three canals; this is the water that has not yet been allocated to a canal. The second argument of the minimum function is the historical allocation for the fourth canal. The minimum function yields the smaller value between the two. Therefore, if the amount of water that has not yet been allocated to a canal exceeds the historical allocation to that canal, the canal will receive its historical water allocation, and water will be available for the next downstream canal.

2.1.2 Upstream Bears Risk of Shortage

Under this institutional arrangement, the seven upper canals deliver the seven lower canals' entire water allocation before receiving any water. After deliveries to the lower canals are secured, all upper canals share remaining water supplies proportionally. This is performed by the function:

$$\min(TotalWaterSupply, HistoricalWaterAllocation_{i=14}) \tag{3}$$

for the most downstream canal,

$$\min(TotalWaterSupply - \sum_{n=1}^{i-1} WaterAlreadyAllocated_i, HistoricalWaterAllocation_i) \tag{4}$$

for second through the seventh most downstream canals (canals 13 through 7), and:

$$HistoricalCanalAllocation_i \cdot \frac{(BasinInflow - \sum_{i=7}^{14} WaterAlreadyAllocated_i)}{MaxTotalBasinInflow - \sum_{i=7}^{14} MaxDownstreamAllocation} \tag{5}$$

for the seven upstream canals. This multiplies the historical canal allocation for each of the seven upstream canals by the ratio of the amount of water that is allocated to the downstream canals and the maximum amount of water that can be allocated to these canals.

Therefore, the downstream canals receive water in the same manner as the Downstream Priority allocation, and the upstream canals share the water that remains after the downstream canals have received their water allocation in proportion to their historical water allocations.

2.1.3 Downstream Bears Risk of Shortage

Under this arrangement for the Balkh Basin, the seven upper canals receive their entire water allocation as a group before the seven lower canals receive any water. After deliveries to the upper canals are secured, all lower canals share the remaining water supplies proportionally. This is performed by the function:

$$\min(TotalWaterSupply, HistoricalWaterAllocation_{i=1}) \tag{6}$$

for the first canal,

$$\min(TotalWaterSupply - \sum_{n=1}^{i-1} WaterAlreadyAllocated_i, HistoricalWaterAllocation_i) \tag{7}$$

for the second through seventh upstream canals (canals 2 through 7), and:

$$HistoricalCanalAllocation_i \cdot \frac{(BasinInflow - \sum_{i=1}^{7} WaterAlreadyAllocated_i)}{MaxTotalBasinInflow - \sum_{i=1}^{7} MaxUpstreamAllocation} \tag{8}$$

for the seven downstream canals. This multiplies the historical canal allocation for each of the seven downstream canals by the ratio of the amount of water that is allocated to the upstream canals and the maximum amount of water that can be allocated to these canals.

This functions in the same manner as the previous arrangement, but the upstream canals share the water that remains after the downstream canals take their historical water allocation proportionally.

2.1.4 Proportional Sharing

Under this arrangement, each canal shares the burden of water shortages proportionally. For example, under this allocation arrangement, a 10 percent reduction in total basin inflow results in a 10 percent reduction in flow to each canal. This is performed by the function:

$$HistoricalCanalAllocation_i \cdot \frac{BasinInflow}{MaxTotalBasinInflow} \tag{9}$$

This function multiplies each canal's historical water allocation by the ratio of the current basin inflows and a full basin inflow (5,253 paikals). This produces a proportional sharing of the available water supply by all of the canals.

2.1.5 User Inserted Allocation

The user is able to insert any possible allocation arrangement using the User Inserted Allocation column found in the Scenario Page tab. The values that the user inserts for each canal are interpreted as the priority that each canal receives in receiving water. A canal that has a 1 in this column will be given the first priority, and a canal with a 14 in this column will be given the fourteenth or last priority. Any values that are tied, that is canals that have the same value in this column, will share water proportionally at the same priority.

The values for the priority given to canals in the default water allocations (the previous four water allocations described above) are provided in columns M through P on the Scenario Page tab for the user's reference. After the user has inserted the desired water allocation scheme, the water allocations for each canal are then calculated using the structure in the Custom Lookup tab. For documentation on how this Custom Lookup tab determines the water allocation for each canal can be found in the document "Excel Documentation for the Balkh River Decision Tool".

2.2 Optimization Model

The optimization model used for this decision support tool is a linear programming model. The optimization model uses the data that is inserted by the user on the User Inputs tab. The user inserts cropland limits, labeled $CroplandLimits_{ij}$, which are the maximum number of hectares that are appropriate for planting the j^{th} crop in the i^{th} canal . Also entered by the user are the cost and return budgets by canal and crop. The net returns per hectare for each crop and canal, labeled $NetReturn_{ij}$, is calculated from the cost and return budgets as $Price_{ij} \cdot Yield_{ij} - Cost_{ij}$. The user also inserts lower bounds, labeled $LowerBounds_{ij}$, which are the minimum number of hectares that must be planted for each crop by canal.

The model produces a food-secure level of wheat production in each canal if sufficient water is available before it produces commerically valued crops. Wheat production is not optimized by the linear programming model, but rather is entered as a constraint. The user inserts the minimum level of wheat production required for food security, which is calculated as the product of the cereal food requirements in kilograms per capita per year, the average number of persons per farm household and the number of farms. This is increased by the percentage of the wheat harvest that is withheld for planting in the next season. The amount of hectares required to produce this level of wheat production is then calculated by dividing the required production by the average yield per hectare. This number of hectares is entered into the model by the following logic:

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    if $TotalLandIrrigated_i \geq WheatLandRequired_i$, then plant $WheatLandRequired_i$
else if $TotalLandIrrigated_i < WheatLandRequired_i$, then plant $TotalLandIrrigated_i$ in wheat
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Which states: if the total land that is irrigated in a particular canal exceeds or is equal to the amount of land required to be brought into wheat production, then the model plants the required amount of wheat production. Or else, if the total land that is irrigated is less than the amount of land required to be brought into wheat production, then the model brings the total area of land that is irrigated into wheat production. If the second condition is true, no land will be allocated to non-wheat crops. This code is run for all 14 canals and all of the five water sharing allocations. These formulas can be found in column P of the tabs that display the results for each allocation arrangement, labeled with the names of the allocation arrangements.

After wheat is produced, the crop mix is chosen by the following linear programming model: Let: i be the set containing the 14 canals, j be the set of commerically valued crops, x_{ij} be the number of hectares of each crop planted in each canal and $NetReturn_{ij}$ be the net return from one hectare of each crop for each canal.

the objective function is then:

$$\max \sum_{ij} NetReturn_{ij} \cdot x_{ij} \tag{10}$$

This finds the optimum level of the endogenous number of hectares of each crop, x_{ij} in each canal. The objective function is subject to the following constraints:

$$x_{ij} \geq LowerBound_{ij} \tag{11}$$

$$x_{ij} \leq CroplandLimts_{ij} \tag{12}$$

$$x_{ij} \geq 0 \tag{13}$$

Equation 11 states that the amount of each crop planted must be equal to or less than the maximum number of hectares that are suitable for that crop in each canal. Equation 12 states that the amount of each crop planted must equal or exceed the user-inserted minimum level of crop production for each crop in each canal. Equation 13 is a sign restriction that states that negative values for crop production are not feasible.

2.3 Mechanics of the Optimization

The optimization is run seperately for each of the five water allocation arrangements described in section 2.1 above. As stated previously, the model first plants the user-defined level of wheat production. The next step in the optimization is to plant the crops required by the user-inserted minimum level of crop production for each crop in each canal defined by $LowerBound_{ij}$. Water that is available in excess of that used to produce wheat and to satisfy the lower bounds is planted with the crop mix that maximizes total basin income.

The first step of the optimization is to determine the crop in each canal whose net return is the highest. The model then plants as much of this crop as possible. If sufficient water is available, the model will plant the number of hectares appropriate for that crop described by $CroplandLimits_{ij}$. If less water is available than sufficient to plant this number of hectares, the model produces as many hectares as possible given the remaining water.

If water remains after the maximum amount of the highest valued crop has been planted, water is then allocated to the second highest valued crop in each canal by the same logic. The model continues to plant crops by this logic until all of the water has been used in crop production, or no further postive returns are possible through crop production.