Explaining Agricultural Water Use Behavior for Policy Analysis:
An Application of Positive Mathematical Programming

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This handout explains the water use behavior of a profit-maximizing irrigation farmer. It shows how to explain current farm income, crop choice, water use, and crop yields, and impacts on each associated with a range of farm and water policies. It rests on discovering two important parameters of the farmer's profitability function. We begin by assuming that any crop's yield declines with larger amounts of land in production, as the most suitable land for each crop is used first:

$$(1)\ Yield = B_0 + B_1 * Land$$

where $B_0$ and $B_1$ are parameters. $B_0$ is the crop yield for the first unit of land in production, and $B_1$ is a negative coefficient, showing the negative impacts of additional land on average crop yield. We expect $B_1$ to be negative from the principle that the best land is planted for any crop before more marginal land enters production. The total water demand for a given crop in production equals the irrigation required per unit land (for full irrigation) times the amount of land in production:

$$(2)\ Water = W_0 * Land$$

where *Water* is total water applied, $W_0$ is the (constant) irrigation required per unit land, *Land* is total land in production. So for any crop, total water demanded is proportional to total land in production.

A profit maximizing farmer chooses an amount of *Land* in production and supporting water that maximizes the difference between total revenues and total cost. The farmer's profit is:

$$(3)\ \pi=[Price_c*Yield - (Cost + Price_w*W_0)]*Land$$

where $\pi$, profit, equals profitability per unit land multiplied by *Land*. Profitability per unit land, the term inside the square brackets, is crop price, $Price_c$, multiplied by *Yield* minus total production cost per unit land. Production costs are non-water cost plus the cost of water, $Price_w$ times $W_0$. $Price_w$ is the water price.

We re-write equation (3) to express a given crop's profitability based on total water applied, *Water*. It replaces land in production with water needed to irrigate the land. Combining (1), (2), and (3), lets us express total profitability as:

$$(4)\ \pi = \left\{[Price_c * (B_0+B_1W_0*Water)] - (Cost + Price_w*W_0)\right\} * Water*W_0$$

While messy, equation (4) expresses profitability in terms of total water use. It shows that the farmer makes choices on the total amount of water to allocate to each crop. Crops not planted use no water. Increases in crop price and yield increase profitability, while increases in the cost of production and the price of water reduce profitability.

The farmer's profit maximizing behavior can be summarized by differentiating the profit function with respect to additional water used. Profit maximization occurs when the profit function is at a maximum with respect to water use. This occurs when the slope of the profit function with respect to water use equals zero. Farmers maximize profits by adding more water until nothing more is added to profit:

$$(5)\quad \frac{\partial \pi}{\partial Water} = \left[\frac{1}{W_0}\right] * Price_c\left[yield - cost - Price_w(W_0) - B_1 * Water\right] = 0$$

Requiring buckets of patience, rearranging terms lets us express the irrigator's demand for water in terms of the price of water charged, $Price_w$

$$(6)\quad Price_w = \left[\frac{1}{W_0}\right][Price_c(Yield - B_1 Water) - Cost]$$

The irrigator has no control over water's price, but only can react to it. Equation (6) states that the irrigator expands water use until the right hand side term equals the water price. He has control over the term, Water, total water use, which also affects yield. That right side term is the economic value of the additional water applied to crops. It's known as the value of the marginal product of water.

We observe the behavior of irrigators who maximize their profits from our data on $Price_w$, $Price_c$, Yields, Cost, $W_0$, and Water. We do not directly see either two parameter of the yield function, $B_0$ and $B_1$.

Irrigation researchers need to calculate what the two parameters of the yield functions, $B_0$ and $B_1$ . It's based the observed behavior of irrigators who have maximized their profits. They are:

$$(7)\quad B_1 = \frac{W_0[Price_w * W_0 - (Price_c * Yield - Cost)]}{Water * Price_c}$$

and

$$(8)\quad B_0 = Yield - B_1\left[\frac{Water}{W_0}\right]$$

These two calculations allow us to calculate the crop yield function that must have produced the crop irrigation and other irrigator behavior we actually saw resulting from profit maximization. Inserting those B0 and B1 into the yield function allows us to express the complete quadratic profit function whose maximization produced the behavior we saw. It is an example of the idea of "positive mathematical programming," coined by Howitt (1995). It also allows us to conduct a number of policy experiments that would affect yields, water supply, input use, and environmental quality.