

Contents

1	Motivation	2
2	Sets	2
3	Data	3
4	Variables (Unknowns)	4
5	Equations	4
	5.1 Water Applied	4
	5.2 Water Depleted	5
	5.3 Land In Production	5
	5.4 Crop Yields	6
	5.5 Fiscal Outcomes	6
	5.6 Income	6
6	Bounds	7
	6.1 Water Applied	7
	6.2 Water Depleted	7
	6.3 Land in Production	8
7	Objective Function	8

Mathematical Documentation, Irrigation Water Policy Model

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1 Motivation

This mathematical documentation presents the essential elements of the Irrigation Water Policy Model (IWPM) developed at the New Mexico State University College of Agricultural, Consumer, and Environmental Sciences in Las Cruces, New Mexico, USA. Additional details and the model's GAMS code are posted on the web at <http://agecon.nmsu.edu/fward/water/>.

While this model and its documentation were originally developed for application to the Lower Rio Grande Project in North America, it is adaptable to the hydrology, land use patterns, irrigation, economics, and institutions of any basin. The essential principle of the hydrology component of model is mass balance, both for surface and groundwater flows, water diversions and water depletions for use in irrigated agriculture.

Other important variables tracked include crop mix, crop yields, land in production, and farm income associated with various policies such as public subsidies of water conserving technology under various water supply scenarios brought about from upstream deliveries, drought, climate change.

The model structure is defined below. That structure includes the model's sets (dimensions), data, variables, and equations.

2 Sets

i resource / land, water /

j crop / alfalfa, cotton, lettuce, onions, green_chile, red_chile, pecan /

k technology / flood, drip /
 n scenario / 100_pct, 80_pct, 60_pct, 40_pct, 20_pct, 00_pct /
 s subsidy / 0, 100/
 w source /sw, gw /

3 Data

All of the following data terms end in _p to distinguish them from variables (unknowns).

YIELD_p(j,k,w) Crop yields (tons/acre) (2010 data)
 PRICE_p(j,p) crop prices from enterprise budgets (\$ per ton)
 RES_PRICE_p(i,j,k,w) resource price (\$/af for water, \$/acre for non-water)
 LAND_TOT_p(j) base land observed in production by crop
 LAND_p(j,k) total land by crop and technology for base year
 TOT_LAND_p total land summed over crops and technologies
 Bw_apply_p(j,k) water application rate (feet depth per season)
 Bw_dep_p(j,k) water depletion rate (ET)

Annpayment_p Annualized capital cost of converting to drip system
 subsidy_p(s) drip irrigation subsidy proportion
 nres_price_p(i,j,k,s,w) net resource price per unit adjusted by drip subsidies
 tx_cost_p (i,j,k,s,w) taxpayer cost of subsidy per acre

Cost_acre_p (j,k,s,w) costs per acre for all inputs derived from input prices
 Netrev_acre_p (j,k,s,w) net revenue per acre

B0_p(j,k) intercept term in crop-water prodn fn vmp of water = water price (PMP)
 B1_p(j,k) linear term in crop-water prodn fn, Ricardian rent (Dagnino-Ward 2012)
 RHS_wdep_p (n) water depletion supplies available by drought and policy scenario

RHS_land_p total observed land in production

4 Variables (Unknowns)

All the following variable terms end in _v, to distinguish them from data.

Yield_v (j,k,n,s,w) yield per acre will vary from base observed levels (tons/ac)

wat_apply_v(n,s,w) water applications per acre (af/ac)

wat_dep_v (n,s,w) water depletion per acre - ET - (af/ac)

t_wat_app_v(n,s) total water applied summed over sources (af)

t_wat_dep_v(n,s) total water depleted summed over sources (af)

wat_app_ac_v(n,s) acre feet per acre water applied av over land - NM OSE 2010 (af/ac)

Tot_land_v(n,s) total land in production (ac)

land_pecan_v(n,s) total land in pecan production (ac)

Grossrev_v (j,k,n,s,w) Gross revenue (\$/ac)

Netrev_v (j,k,n,s,w) Net revenue (\$/ac)

TX_cost_v(j,n,s,w) taxpayer program cost (\$)

Income_jk_v(j,k,n,s,w) farm income disaggregated (\$)

Income_w_v (n,s,w) total farm income by water source (\$)

Income_v (n,s) Total farm income (\$)

Land_v (j,k,n,s,w) land in production disaggregated (must be > 0)

5 Equations

5.1 Water Applied

Total water applied to crop irrigation is measured as:

$$(1) \quad \text{wat_apply_v}(n,s,w) = \sum((j,k), \text{Land_v}(j,k,n,s,w) * \text{Bw_apply_p}(j,k))$$

Total water applied to crops summed over the sources of groundwater and surface water is:

$$(2) \quad \text{t_wat_app_v}(n,s,) = \sum(w, \text{wat_apply_v}(n,s,w));$$

Average water applications per acre over all land in production is:

$$(3) \quad \text{wat_app_ac_v}(n,s,) = \text{t_wat_app_v}(n,s,) / [1 + \text{tot_land_v}(n,s,);$$

Water is administered by the New Mexico State Engineer's Office, and an upper bound of total applications from all sources cannot exceed 4.5 acre feet per acre. Where the small constant 1 is included in the denominator to avoid dividing by zero when no land is in production.

5.2 Water Depleted

Total water depleted in irrigation is measured as:

$$(4) \quad \text{wat_dep_v}(n,s,w) = \sum ((j,k), \text{Land_v}(j,k,n,s,w) * \text{Bw_dep_p}(j,k))$$

Total water depleted by crops summed over sources of groundwater and surface water is:

$$(5) \quad \text{t_wat_dep_v}(n,s,) = \sum(w, \text{wat_dep_v}(n,s,w));$$

5.3 Land In Production

Total land in production, summed over all crops, technologies, and water sources is:

$$(6) \quad \text{Tot_land_v}(n,s) = \sum((j,k,w), \text{Land_v}(j,k,n,s,w)).$$

Total land in production for pecan orchards is:

$$(7) \quad \text{Land_pecan_v}(n,s) = \text{sum}((k,w), \text{Land_v}('7-pecan',k,n,s,w));$$

This land is constrained to never fall below base current levels (23,000 acres) because of the very lucrative economic profitability that would be lost for future years if inadequate amounts of water were applied to those orchards.

5.4 Crop Yields

Observed crop yields under surface water decline with expanded acreage, based on the principle of Ricardian rent. This equation uses positive mathematical programming to assure that small water supply scenarios or small policy changes produce small results.

$$(8) \quad \text{Yield_sw_v} (j,k,n,s) = B0_p(j,k) + B1_p(j,k) * \text{sum} (w, \text{Land_v}(j,k,n,s,w)).$$

Yields from crops irrigated by groundwater pumping are lower than surface yields, ranging from 75% to 90% of surface water yields:

$$(9) \quad \text{Yield_gw_v} (j,k,n,s,p,c) = \text{yield_salinity_p}(j) * \text{Yield_v}(j,k,n,s,'1-sw')$$

5.5 Fiscal Outcomes

$$(10) \quad \text{TX_cost_v} (j,n,s,w) = \text{sum}(k, \text{Land_v}(j,k,n,s,w) * \text{tx_cost_p}('land',j,k,s,w)).$$

5.6 Income

$$(11) \quad \text{Grossrev_v}(j,k,n,s,w) = \text{Price_p}(j) * \text{Yield_v} (j,k,n,s,w)$$

Net revenue per acre equals price times yield minus costs of production including all water costs:

$$(12) \quad \text{Netrev_v} (j,k,n,s,w) = \text{Price_p}(j) * \text{Yield_v}(j,k,n,s,w) - \text{Cost_acre_p}(j,k,s,w)$$

Income, tracked in various ways is defined below as:

$$(13) \quad \text{Income_jk_v}(j,k,n,s,w) = \text{Land_v}(j,k,n,s,w) * \text{Netrev_v}(j,k,n,s,w)$$

$$(14) \quad \text{Income_w_v}(n,s,w) = \text{sum}(j,k, \text{Land_v}(j,k,n,s,w) * \text{Netrev_v}(j,k,n,s,w))$$

$$(15) \quad \text{Income_v}(n,s) = \text{sum}(w, \text{Income_w_v}(n,s,w)).$$

6 Bounds

6.1 Water Applied

The NM State Engineer's office limits total water applications to 4.5 acre feet per acre, summed over surface and groundwater source. This amount is averaged over all lands in production.

$$(16) \quad \text{wat_app_ac_v.up}(n,s,p,c) = 4.5;$$

Total surface water available for application is defined by a given year's runoff combined with a rule known as the Rio Grande Compact that defines how headwater river flows are shared among the three states, Colorado, New Mexico, and Texas.

$$(17) \quad \text{wat_apply_v.up}(n,s,p,c,'1\text{-sw}') = \text{RHS_wapply_p}(n);$$

This constraint enables the model to be run under a range of water supply shortages from a full supply to an extreme drought of zero surface water supplies.

6.2 Water Depleted

An upper bound on total surface water depletions is:

$$(18) \quad \text{wat_dep_v.up}(n,s,p,c,'1\text{-sw}') = \text{RHS_wdep_p}(n);$$

This upper bound is established as a way to protect senior water right holders, when drip irrigation is

installed for which depletions are typically higher than under flood irrigation. Groundwater pumping is not directly regulated other than the 4.5 acre feet per acre of total use described above. It is limited more by the cost of pumping and by the high salinity that reduces crop yields.

6.3 Land in Production

Total land in production cannot exceed base acreage, about 90,000 acres, of which about 2/3 are accounted for in the current study.

$$(19) \quad \text{tot_land_v.up}(n,s,p,c) = \text{RHS_land_p.}$$

Total land in pecan production cannot fall below base observed levels (23,000 acres).

$$(20) \quad \text{land_pecan_v.lo}(n,s,p,c) = \text{Land_tot_p('7-pecan')};$$

There is no drip irrigation under surface irrigation permitted when surface supplies falls below 40% of full surface supply. This is because the canals have too little water in them to be reliable as a source for drip irrigation.

$$(21) \quad \text{Land_v.fx}(j, '2\text{-drip}', '5\text{-}20\text{_pct}', s,p,c, '1\text{-sw}') = 0;$$

$$(22) \quad \text{Land_v.fx}(j, '2\text{-drip}', '6\text{-}00\text{_pct}', s,p,c, '1\text{-sw}') = 0;$$

7 Objective Function

The model objective is to maximize net income defined in equation (15). The income maximization occurs for each of the six water supply scenarios and each of the two drip irrigation conversion subsidy levels.